Journal of Operation and Automation in Power Engineering
Vol. 5, No. 2, Dec. 2017, Pages: 139-149
http://joape.uma.ac.ir

Demand Response Based Model for Optimal Decision Making for Distribution Networks

M. Kazemi-Khafri, A. Badri, A. Motie-Birjandi*

Faculty of Electrical Engineering
Shahid Rajaee Teacher Training University, Tehran, Iran

Abstract*-In this paper, a heuristic mathematical model for optimal decision-making of a Distribution Company (DisCo) is proposed that employs demand response (DR) programs in order to participate in a day-ahead market, taking into account elastic and inelastic load models. The proposed model is an extended responsive load modeling that is based on price elasticity and customers' incentives in which they participate in demand response program, voluntarily and would be paid according to their declared load curtailment amounts. It is supposed that DisCo has the ability to trade with the wholesale market and it can also use its own distributed generation (DG), while decision making process. In this regard, at first, DisCo's optimization frameworks in two cases, with and without elastic load modelings are acquired. Subsequently, utilizing Hessian matrix and mathematical optimality conditions, optimal aggregated load curtailment amounts are obtained and accordingly, individual customer's load reductions are calculated. Furthermore, effects of DG contributions and wholesale electricity market are investigated. An IEEE 18 bus test system is employed to obtain the results and show the accuracy of the proposed model.*

Keyword: Decision-making, Distribution company, Demand response, Load elasticity, Distributed generation.

NOMENCLATURE

N_d	Number of customers
N_g	Number of DG units
j	Number of DG units
i	Index for DG unit
k	Index for connection points to wholesale market
$C_g^i(P_g^i(t))$	Operation cost of ith DG unit
$C_{DR}^j(DR^j(t))$	Cost of implementing demand response
$C_w(W(t))$	Disco's cost for purchasing energy from wholesale market at time t
$C_d^j(D_0^j(t))$	Disco's revenue from selling energy to jth customer at time t
$D_0^j(t)$	Fixed (Initial) consumption of jth customer at time t for inelastic (elastic) load modeling at time t
$D^j(t)$	Updated jth customer's consumption at time t
$DP(t)$	Amount of DR incentives at time t
$DR^j(t)$	Amount of jth customer's reduced demand at time t
E	Self elasticity
$E(t,t')$	Mutual elasticity
$P_g^i(t)$	Amount of ith DG unit contribution at time t
$RD(t)$	Customer's total reduced demand at time t
$S(t)$	Binary variable for DG status at time t, 1 if DG is on, 0 otherwise
STC	Start up cost of DG
$b(t)$	Binary variable for DG start up at time t, 1 if DG is start up, 0 otherwise
a_i, b_i	Cost coefficients of DR programs
$\alpha_i, \beta_i, \gamma_i$	Cost coefficient of ith DG unit
$\rho_{0d}(t)$	Initial price of selling energy at time t for elastic load modeling
$\rho_d(t)$	Price (updated price) of selling energy for inelastic (elastic) load modeling at time t
$\rho_w(t)$	Amount of purchased (sold) energy from (to) wholesale market at time t
$W(t)$	Amount of purchased (sold) energy from (to) wholesale market at time t

Received: 24 Jun. 2016
Revised: 5 Sept. 2016 and 10 Jan. 2017
Accepted: 23 Jan. 2017
∗Corresponding author:
E-mail: a_badri73@yahoo.com (A. Badri)
Digital object identifier: 10.22098/joape.2017.2475.1214

1. INTRODUCTION

Demand response provides an opportunity for consumers to play a significant role in the operation of the electric grid by reducing or shifting their electricity usage during peak periods in response to time-based rates or other forms of financial incentives. Demand response programs are being used by electric system planners, operators and consumers as resource options for balancing supply and demand and reducing electrical consumption costs. Such programs can reduce the cost of electricity in wholesale markets, and in turn, lead to lower retail rates [1]. Demand reduction can be interpreted as a power plant except that, is has much lower marginal cost in comparison to real power production units.Load reduction techniques are known as demand response programs which may be time based or incentive based, each has a specific role on system characteristics [2,3].

Time-based programs include time of use, real time pricing, and critical peak pricing strategies. In these programs, the electricity price changes for different periods, so customers should adjust their consumption according to the time and associated tariffs [4]. On the other hand, incentive- based programs consist of direct load control, emergency demand response program, interruptible/curtailable service, and capacity market program. In these programs, customers are being encouraged with independent system operator (ISO) or local utility to moderate their consumption [4]. Demand responseis a useful tool for the independent system operator, which can be activated within a short time in critical system conditions. In [5], an economic model of price/incentive responsive loads for demand response has been developed based on the concept of price elasticity of demand and customers' benefit function.The focus is on direct load control and emergency demand response programs. Economic models of time based and incentive-based programs have been addressed in many researchers in recent years. Reference [6] has presented an economic model of price responsive loads based on constant value of price elasticity. In Ref. [7], economic models of responsive loads have been derived and their impacts on system characteristics are discussed. A dynamic DR pricing is presented in [8] in which interaction between the load serving entity and its customers is formulated as a bilevel optimization problem where the load serving entity is the leader and DR aggregators are the followers. In Ref. [9] a profit-maximization-based pricing optimization model for the demand response management with customer behavior learning is proposed. An incorporated emergency demand response program and unit commitment model is presented in [10] in which impacts of demand response model on unit participation factors are investigated. A method for determining the optimal incentives in incentive-based DR programs is proposed in [11]. A cost-benefit based demand response in presence of renewable resources is presented in [12] in which impacts of strategies on load shape, benefits of customers and the reduction of energy consumption are inspected. Despite other papers that deal with residential and commercial consumers, Ref. [13] provides an approach for time of use demand response for industrial manufacturing systems under production target constraints. Accordingly, electricity related costs are integrated in production system modeling. Ref. [14] proposes a decentralized framework in which the aggregator seeks to maximize its profits while the consumers minimize their costs in response to time-varying prices, and additional incentives provided to mitigate potential overloads in the distribution system. Some studies have proposed decision making frameworks for distribution companies or retailers to purchase the electricity and provide for the end users; however, not paid enough attention in applications of demand response programs [15]. The effect of demand side management on load elasticity and market power exerted by generation companies is represented in [16]. In Ref. [17] a tool is described from aggregator's perspective in order to forecast the load demand response of subscribed customers and minimize their electricity bills. This tool allows aggregator to choose the most appropriate control strategy for market participants. The integration of responsive loads in distribution expansion planning is investigated in [18]. The model is from utility owner's perspective and the objective function is minimizing total cost of line installations, maintenance, energy losses, as well as system reliability.

In this paper, a novel mathematical framework is proposed for decision making of DisCos in which elastic and inelastic responsive load models are utilized to obtain the most beneficial strategies for distribution companies. The proposed model is based on an incentive based load control in which customers participate in demand response program voluntarily and would be paid according to their declared load curtailment amounts; nevertheless, are not penalized otherwise. Furthermore, impacts of upper level trading as well as owned DG contributions are taken into account. Accordingly, a combinatorial objective function is proposed to obtain DisCo's maximum profit. For the sake of reality, customers' participation factors are also considered in proposed modeling. The main innovation of this study in comparison to similar ones is presenting a novel mathematical framework for DisCo's optimal decision making rather than employing optimization algorithms. Furthermore, unlike previous studies that consider predetermined load reductions and incentives for the subscribers, here, optimal load curtailment amounts as well as corresponding incentive prices that result in DisCo's maximum payoff are derived. In this regard, at first DisCo's optimization frameworks in two cases with and without elastic load modelings are presented. Subsequently, utilizing Hessian matrix and mathematical optimality conditions, optimal aggregated and individual load reductions as well as corresponding hourly incentive prices are obtained. In addition, effects of DG contributions and upstream wholesale electricity market are investigated. An IEEE 18 bus test system is employed to obtain the results and show the accuracy of the proposed model. It should be noted that here a single period demand response modeling is assumed in which consumers face load alleviations when called by DisCo. Therefore, self elasticity coefficients are considered and the impact of the mutual elasticity is neglected. The rest of the paper is organized as follows:

In section two, the proposed mathematical framework as well as corresponding optimality conditions are represented. Subsequently, mathematical expressions for DisCo's optimal decision making in presence of inelastic and elastic load modelings are derived. A case study is presented in section three and finally section four provides the conclusion.

2. Problem formulation

This section describes how to prepare each part of the final manuscript more specificallyEach paper size should be A4 and the margins should be 1.33 inches from top and 0.98 inches from bottom, right, and left sides, respectively. The represented framework is a decision making model for a distribution company that includes a combinatorial objective function in which owned distributed resources as well as elastic and inelastic responsive loads are employed as some tools to obtain the most profitable strategy for DisCo. For obtaining more realistic results, DisCo's trading with wholesale market is also taken into account. Subsequently, proposed objective function would be as Eq. (1) :

$$Profit = \max\left\{ \sum_{j=1}^{N_d} C_d^j(D_0{}^j(t)) - C_w(W(t)) - \sum_{j=1}^{N_d} C_{DR}^j(DR^j(t)) - C_g(P_g(t)) \right\} \tag{1}$$

In which the first term represents DisCo's revenue based on energy consumption and related selling price as Eq. (2):

$$C_d^j(D_0{}^j(t)) = D_0{}^j(t) \times \rho_d(t) \tag{2}$$

The second term refers to purchasing (selling) energy from (to) wholesale market based on traded energy with upstream market price as below:

$$C_w(W(t)) = \rho_w(t) \times W(t) \tag{3}$$

The third term states the imposed cost to distribution company arising from executing demand response that is calculated as below. This cost is due to incentives that should be paid to those customers participating in load reduction program. As shown in Eq. (4) the aggregated payable amount exceeds selling price in order to encourage customers to attend DR programs.

$$\sum_{j=1}^{N_d} C_{DR}^j(DR^j(t)) = (DP(t) + \rho_d(t)) \times \sum_{j=1}^{N_d} DR^j(t) \tag{4}$$

and finally Eq. (5) illustrates ith DG operation cost.

$$C_g^i(P_g^i(t)) = (\alpha_i P_g^i(t)^2 + \beta_i P_g^i(t) + \gamma_i)S(t) + STC.b(t) \tag{5}$$

Substituting, Eqs. (2) to (5) in Eq. (1) the proposed objective function for DisCo is derived as below:

$$\begin{aligned} Profit = \max\Big\{ & \sum_{j=1}^{N_d} D_0{}^j(t) \times \rho_d(t) - \rho_w(t) \times W(t) \\ & - (DP(t) + \rho_d(t)) \times \sum_{j=1}^{N_d} DR^j(t) - \\ & (\sum_{i=1}^{Ng} (\alpha_i P_g^i(t)^2 + \beta_i P_g^i(t) + \gamma_i)S(t) + STC.b(t)) \Big\} \end{aligned} \tag{6}$$

As it is appear from Eq. (6), demand side management (DSM) has been considered as a main applicable tool for DisCo's decision making. Therefore, in this regard an economic load model which represents the change of customer's demand with respect to change of electricity price and incentives is developed here. Due to variety of load modelings in DSM programs and for the sake of reality the proposed mathematical model is categorized based on inelastic and elastic load modelings that will be represented as follows :

2.1. Inelastic load modeling

Here it is assumed that load is inelastic so that the customer load amount is unchanged with respect to price volatility; however, there would be some changes due to applying incentives to responsive loads. Assuming customers' aggregated initial and reduced load amounts as Eqs. (7) and (8):

$$\sum_{j=1}^{N_d} D_0{}^{j}(t) = D_0 \quad (7)$$

$$\sum_{j=1}^{N_d} DR^{j}(t) = RD(t) \quad (8)$$

Eq. (6) can be rewritten as :

$$\begin{aligned} \Pr ofit = \max\{ & D_0(t)\times\rho_d(t) - \rho_w(t)\times W(t) - \\ & (DP(t)+\rho_d(t))\times RD(t) - \\ & (\sum_{i=1}^{Ng}(\alpha_i P_g{}^{i}(t)^2 + \beta_i P_g{}^{i}(t) + \gamma_i)S(t) + STC.b(t))\} \end{aligned} \quad (9)$$

Considering various load resources the load balance equation can be represented as below. In the other words Eq. (10) represents the amount of energy that should be traded with upstream market at time t. It is noticeable that all DG units are assumed to be owned by the same DisCo.

$$W(t) = D_0(t) - RD(t) - \sum_{i=1}^{Ng} P_g^i(t) \quad (10)$$

Substituting Eq. (10) in Eq. (9) we have:

$$\begin{aligned} \Pr ofit = \max\{ & D_0(t)\times\rho_d(t) - \rho_w(t)\times(D_0(t) \\ & - RD(t) - \sum_{i=1}^{Ng} P_g{}^{i}(t)) - (DP(t)+\rho_d(t))\times RD(t) \\ & -(\sum_{i=1}^{Ng}(\alpha_i P_g{}^{i}(t)^2 + \beta_i P_g{}^{i}(t) + \gamma_i)S(t) + \\ & STC.b(t))\} \end{aligned} \quad (11)$$

Let a linear relationship exists between DR incentive price and j th customer's reduced load amount for each time as Eq. (12) [19]. Actually, this equation shows customer preference to participate in DR program. As indicated, it depends on a_i, b_i coefficients such that the lower a_i and b_i means the higher customer tendency to execute DR programs.

$$DR^{j}(t) = \frac{DP(t) - b_j}{a_j} \quad (12)$$

Thus, taking into account Eq. (8), amount of DR incentive price can be derived as Eq. (13):

$$DP(t) = \frac{RD(t) + \sum_{j\neq i}^{N_d} \frac{b_j}{a_j}}{S} \quad (13)$$

In which:

$$S = \sum_{j=1}^{N_d} \frac{1}{a_j} \quad (14)$$

In Eq. (13), RD (t) is the aggregated demand reduction for all participants that in turn can be interpreted as the difference between initial and reduced load amounts of customer j as shown in Eq. (15):

$$D_0(t) - D(t) = RD(t) \quad (15)$$

In above equation D(t) is customers aggregated load amount after implementing DR that is calculated by summation of all individual loads as:

$$\sum_{j=1}^{N_d} D^{j}(t) = D(t) \quad (16)$$

Accordingly, DisCo's optimal objective function can be represented as follows:

$$\begin{aligned} \Pr ofit = \max\{ & A(t)\times D(t) - \frac{D_0{}^{2}(t)}{S} + \\ & \frac{2D(t)\times D_0(t)}{S} + \frac{D_0(t)}{S}\times\sum_{j\neq i}^{N_d}\frac{b_j}{a_j} \\ & - \frac{D^2(t)}{S} + \frac{D(t)}{S}\times\sum_{j\neq i}^{N_d}\frac{b_j}{a_j} + \rho_w(t) \\ & \times\sum_{i=1}^{Ng} P_g{}^{i}(t) - (\sum_{i=1}^{Ng}(\alpha_i P_g{}^{i}(t)^2 + \beta_i P^{i}{}_g(t) \\ & + \gamma_i)S(t) + STCb(t))\} \end{aligned} \quad (17)$$

In which A(t) is the difference between customer selling price and upstream wholesale market price for each hour as Eq. (18):

$$A(t) = \rho_d(t) - \rho_w(t) \quad (18)$$

Considering the objective function is called F, derivation of Eq. (17) with respect to D(t) one can conclude:

$$\frac{\partial F}{\partial D} = A(t) + \frac{2D_0(t)}{S} - \frac{2D(t)}{S} + \frac{1}{S}\sum_{j\neq i}^{N_d}\frac{b_j}{a_j} \quad (19)$$

By equating Eq. (19) to zero, Eq. (20) is obtained that shows customers optimal (alleviated) load amount that results in maximum profit for DisCo.

$$D(t) = \left[\frac{A(t)\times S}{2} + D_0(t) + \frac{1}{2}\sum_{j\neq i}^{N_d}\frac{b_j}{a_j}\right] \quad (20)$$

Similarly, by derivation of Eq. (17) with respect to $P_g{}^{i}(t)$ and equating to zero Eq. (21) is derived as bellow:

$$P_g{}^{i}(t) = \frac{\rho_w(t) - \beta_i}{2\alpha_i} \quad (21)$$

On the other hand, second order derivatives of the objective function with respect to D(t) and $P_g{}^{i}(t)$ are as Eqs. (22) and (23) :

$$\frac{\partial^2 F}{\partial D^2} = -\frac{2}{S} < 0 \tag{22}$$

$$\frac{\partial^2 F}{\partial {P_g}^2} = -2\alpha_i < 0 \tag{23}$$

Accordingly, the Hessian matrix of the optimality problem is obtained as Eq. (24)

$$H = \begin{bmatrix} \frac{\partial^2 F}{\partial D^2} & \frac{\partial^2 F}{\partial D \partial P_g^i} \\ \frac{\partial^2 F}{\partial P_g^i \partial D} & \frac{\partial^2 F}{\partial P_g^{i2}} \end{bmatrix} = \begin{bmatrix} -\frac{2}{S} & 0 \\ 0 & -2\alpha_i \end{bmatrix} \tag{24}$$

Given that the both second order derivatives are negative and since first square determinant of Hessian matrix is negative ($-\frac{2}{S}$) and second square determinant of Hessian matrix is positive ($\frac{4\alpha}{S}$) and according to optimality condition the obtained values for D(t) and P_g^i will maximize the objective function that is desirable from DisCo's point of view.

Obtaining customers' optimal (alleviated) load amount and taking into account aggregated initial load, optimal load curtailment amount is calculated by Eq. (15). Subsequently, optimal load incentive price and individual load curtailment amounts are obtained via Eqs. (13) and (12), respectively. Individual load curtailment amounts represent the curtailment quotas of individual customer participating in DR program.

Finally, amount of DisCo's tradable energy with upstream wholesale market may be obtained by Eq. (10).

2.2. Elastic load modeling

Besides inelastic loads, there may be price responsive loads that are changed based on price fluctuations. Load elasticity is defined as the relationship between load and price variations. A variety of mathematical functions can be used to show the load elasticity model in terms of linear, exponential logarithmic and hyperbolic functions each applicable for different types of customers.

In this case system load would be sensitive to price variations. Accordingly, load elasticity is defined as Eq. (25) [20].

$$E = \frac{\rho_0}{d_0} \cdot \frac{\partial d}{\partial \rho} \tag{25}$$

In those electrical systems that price is time dependant, customers' behaviors may be different with respect to price variations. Some loads such as lighting loads are fixed and cannot be shifted to other periods. These kinds of loads have single period sensitivity and have self elasticity with price. The self elasticity coefficients are always non-positive indicating the higher price leads to the lower consumption.

On the other hand, some loads may be shifted to off-peak periods due to high prices in peak hours. These loads have multiple-period sensitivity and have mutual elasticity with price. The mutual elasticity coefficients are always non-negative indicating that higher price in one period leads to higher consumptions in some other periods.

For 24 hours load variations and assuming one hour for each time span, load elasticity matrix can be written as follows.

$$E = \begin{bmatrix} \xi_{1,1} & \xi_{1,2} & \cdots & \xi_{1,23} & \xi_{1,24} \\ \xi_{2,1} & \xi_{2,2} & \cdots & \xi_{2,23} & \xi_{2,24} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \xi_{23,1} & \xi_{23,2} & \cdots & \xi_{23,23} & \xi_{23,24} \\ \xi_{24,1} & \xi_{24,2} & \cdots & \xi_{24,23} & \xi_{24,24} \end{bmatrix} \tag{26}$$

In which diagonal and non-diagonal elements are self elasticity and mutual elasticity coefficients, respectively. Columns of the matrix show the effects of electricity price on loads within all periods of time. The matrix represents customers to shift their load demands based on price variations such that if the elements above the main diagonal are non-zero it indicates that customers try to consume their loads, in advance in order not to coincide with higher price. On the other hand, if elements below the main diagonal are non-zero it means that customers try to postpone their loads not to face peak price periods. In this study, all loads are assumed to have self elasticity and all mutual elasticity coefficients are zero.

Considering load elasticity equation in Eq. (25), one can rewrite the equation as below:

$$E = \frac{\dfrac{D(t) - D_0(t)}{D_0(t)}}{\dfrac{\rho_d(t) - \rho_{0d}(t)}{\rho_{0d}(t)}} \tag{27}$$

Substituting Eqs. (10), (13), (15) and (27) in Eq. (9) DisCo's objective function would be deduced as Eq. (28):

$$\begin{aligned} Profit = \max\{ & \rho_{0d}(t)\times D(t)+\frac{D^2(t)}{D_0(t)\times E}\times\rho_{0d}(t) \\ & -\frac{D(t)}{E}\times\rho_{0d}(t)-\rho_w(t)\times D(t)+\rho_w(t)\times P(t) \\ & -\left[\frac{D^2(t)}{S}+\frac{D_0^{\,2}(t)}{S}-\frac{2D_0(t)\times D(t)}{S}\right] \\ & +\left[\frac{D(t)}{S}\sum_{j\neq i}^{N_d}\frac{b_j}{a_j}-\frac{D_0(t)}{S}\sum_{j\neq i}^{N_d}\frac{b_j}{a_j}\right]- \\ & (\sum_{i=1}^{Ng}(\alpha_i P_g^i(t)^2+\beta_i P_g^i(t)+\gamma_i)S(t)+STCb(t))\} \end{aligned} \tag{28}$$

Derivation of Eq. (28) with respect to D(t) and equating to zero, one can reach to Eq. (29) that shows customers new consumption, which results in maximum profit for the DisCo.

$$\begin{aligned} D(t)= & \frac{-\rho_{0d}(t)\times D_0(t)\times S\times E+\rho_{0d}(t)\times D_0(t)\times S}{2\rho_{0d}(t)\times S-2D_0(t)\times E} \\ & +\frac{\rho_w(t)\times D_0(t)\times S\times E-2D_0^{\,2}(t)\times E}{2\rho_{0d}(t)\times S-2D_0(t)\times E} \\ & +\frac{-D_0(t)\times E\times\sum_{j\neq i}^{N_d}\frac{b_j}{a_j}}{2\rho_{0d}(t)\times S-2D_0(t)\times E} \end{aligned} \tag{29}$$

Obtaining $D(t)$ and taking into account demand elasticity, the updated energy price offered by DisCo is calculated as below:

$$\rho_d(t)=\frac{(D(t)-D_0(t))\times\rho_{0d}(t)}{E\times D_0(t)}+\rho_{0d}(t) \tag{30}$$

Similarly, derivation of Eq. (28) with respect to $P_g^{\,i}(t)$ and equating to zero the optimal contribution of ith DG unit is obtained as follows:

$$P_g(t)=\frac{\rho_w(t)-\beta_i}{2\alpha_i} \tag{31}$$

On the other hand second order derivatives of the objective function (entitled F) with respect to D(t) and $P_g^{\,i}(t)$ are provided in Eqs (32), (33):

$$\frac{\partial^2 F}{\partial D^2}=\frac{2\rho_{0d}(t)}{D_0(t)\times E}-\frac{2}{S}<0 \tag{32}$$

$$\frac{\partial^2 F}{\partial {P_g^{\,i}}^2}=-2\alpha_i<0 \tag{33}$$

And subsequently, corresponding Hessian matrix would be derived as below:

$$H=\begin{bmatrix}\frac{\partial^2 F}{\partial D^2} & \frac{\partial^2 F}{\partial D\,\partial P_g^{\,i}} \\ \frac{\partial^2 F}{\partial P_g^{\,i}\,\partial D} & \frac{\partial^2 F}{\partial {P_g^{\,i}}^2}\end{bmatrix}= \begin{bmatrix}\frac{2\rho_{0d}(t)}{D_0(t)\times E}-\frac{2}{S} & 0 \\ 0 & -2\alpha_i\end{bmatrix} \tag{34}$$

Again considering that second derivatives with respect to both variables are negative and first and second square determinants of Hessian matrix are negative and positive, the obtained D(t) and $P_g^{\,i}(t)$ will maximize DisCo's objective function.

Having optimal values of D(t) and $P_g^{\,i}(t)$, other unknown variables may be obtained, using Eqs (12) , (13) and (15).

For those category of loads that can be shifted to off-peak periods, mutual elasticity is defined. Accordingly, mutual elasticity of t^{th} period with respect to t'^{th} period is calculated based on Eq. (35) [21].

$$E(t,t')=\frac{\frac{D(t)-D_0(t)}{D_0(t)}}{\frac{\rho_d(t')-\rho_{0d}(t')}{\rho_{0d}(t')}} \tag{35}$$

Similarly, by means of mathematical formulations, it can be concluded that in a multi period model, amount of load in each period depends on price variations in that period and other time periods as well. Eq. (36) shows the customers' updated demand after implementing DR program in a multi period framework.

$$\begin{aligned} D(t)=\sum_{\substack{t'=1\\ t\neq t'}}^{24}(& \frac{-\rho_{0d}(t')\times D_0(t)\times S\times E(t,t')}{2\rho_{0d}(t')\times S-2D_0(t)\times E(t,t')}+ \\ & \frac{\rho_{0d}(t')\times D_0(t)\times S+\rho_w(t')\times D_0(t)\times S\times E(t,t')}{2\rho_{0d}(t')\times S-2D_0(t)\times E(t,t')}) \\ & +\frac{-2D_0^{\,2}(t)\times E(t,t')-D_0(t)\times E(t,t')\times\sum_{j\neq i}^{N_d}\frac{b_j}{a_j}}{2\rho_{0d}(t')\times S-2D_0(t)\times E(t,t')}) \end{aligned} \tag{36}$$

3. Case study

In this section in order to show the accuracy of the proposed model, a case study is implemented. An IEEE 18 bus test system is utilized for this purpose (See Appendix):

Table 1 shows the customers' permissible load reduction amounts as well as corresponding coefficients at each bus.

Table 2 represents price of selling energy to customers as well as price of purchasing energy from wholesale market for each hour. In addition, customers' initial load amounts are also provided in this Table.

As illustrated in some hours, selling price to customers exceeds purchasing price from wholesale market, however, at some other hours (like 13 to 21) it is vice versa. Also distributed generator cost coefficients are represented in Table 3.

Table 1. DR characteristics of individual customers [4]

Bus	a	b	Max DR(MW)	Bus	a	b	Max DR(MW)
1	0.7	2.21	0.366	**10**	0.77	2.28	0.244
2	0	0	0	**11**	0	0	0
3	0.75	1.85	0.821	**12**	0.7	2.29	0.252
4	0.72	2.29	0.244	**13**	0.73	2.29	0.244
5	0.71	2.24	0.315	**14**	0	0	0
6	0.78	2.28	0.244	**15**	0.78	2.27	0.255
7	0.80	2.27	0.244	**16**	0	0	0
8	0.73	2.29	0.230	**17**	0.72	2.33	0.192
9	0.76	2.01	0.594	**18**	0.78	1.92	0.704

Table 2. Hourly initial load amounts and related prices

Hours	$D_0(t)(MW)$	$\rho_d(t)(\$/MWh)$	$\rho_w(t)(\$/MWh)$
1	13.69	38	30
2	12.50	38	29
3	11.61	38	29
4	11.31	38	28.5
5	14.88	38	32
6	15.17	38	32
7	16.37	60	55
8	17.85	60	55
9	20.24	60	55
10	20.83	60	56
11	18.45	67	65
12	16.66	67	67
13	15.47	67	69.5
14	15.17	60	62.4
15	14.88	60	62.3
16	15.77	60	62.4
17	16.07	60	62.3
18	20.53	60	62.2
19	22.91	67	69.7
20	25.00	67	69.5
21	23.21	67	69.6
22	20.24	67	62.3
23	17.85	60	55
24	14.88	60	54

The study is implemented in two scenarios, with inelastic and elastic load modelings, respectively.

In the latter case, it is assumed that loads have single period sensitivity with self elasticity of -0.1 and no mutual elasticity.

Table 3. DG cost coefficients [22]

DG	Max P(MW)	α	β
1	4	0.048	50
2	5	0.060	65
3	5.5	0.055	55
4	7	0.050	60

Based on opimal reduced loads, Fig. 1 illustrates customers' hourly load curtailments for inelastic and elastic load modelings. As it is appear, for inelastic load modeling, customers face load curtailments at hours 13 to 21 since within these periods it is not beneficial for DisCo to purchase energy from wholesale market. Note that during hours 1 to 12 no load curtailment is occured due to relatively low energy purchase prices. In addition, maximum load reductions have been occured at hours 13 and 19 due to relatively high market prices within these periods. For elastic load modeling, subscribers face load curtailment in those periods that it is not beneficial for DisCo to purchase energy from wholesale market. However, unlike inelastic load modeling, here customers experience load reductions from hour 11 and are exposed to their maximum permissible load curtailments within hours 12 to 21. This is because of customers' preferences for load reduction due to increased selling prices within these periods. In this case, it would be more beneficial for DisCo to apply maximum permissible load alleviations, despite paying much more incentives to the curtailed customers.

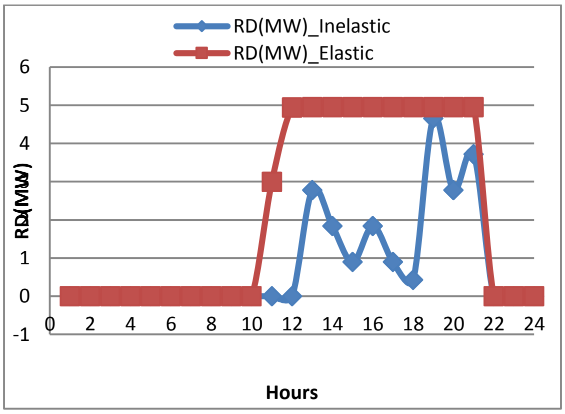


Fig. 1. Customers' total load curtailments for each hour

Accordingly, Fig. 2 illustrates hourly price incentives payable by DisCo to responsive subscribers. As expected, the incentive prices vary in accordance with corresponding load curtailment amounts such that in the first 12 hours no incentive is necessary for inelastic load modeling. However, applying maximum load reduction within hours 12 to 21 in elastic loads, results in maximum incentives for corresponding customers. Note that at hour 11 the incentive price varies proportional to applied load curtailment.

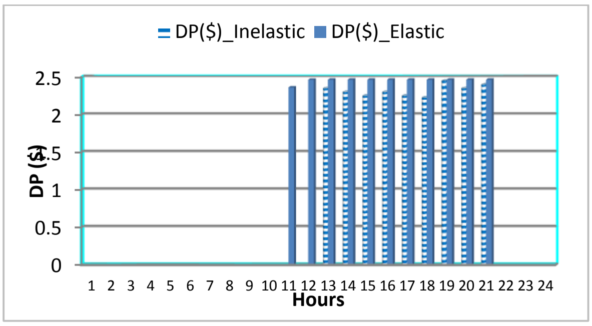


Fig. 2. Hourly incentive prices payable by DisCo to customers

Taking in to account hourly aggregated load curtailments the contribution of each individual customer in providing load mitigation can be extracted. For inelastic load modeling, the quota of each subscriber in total load curtailment amount at hour 20 is illustrated in Fig. 3. As can be seen the lower $a_{i.}$ and $b_{i.}$ coefficients leads to the higher customer preferences to reduce the load. Accordingly, consumers in buses 3, 9 and 18 relatively experience higher load curtailments based on their requests, whereas consumers in buses 4, 8, 13 and especially 17 are exposed to lower load mitigations.

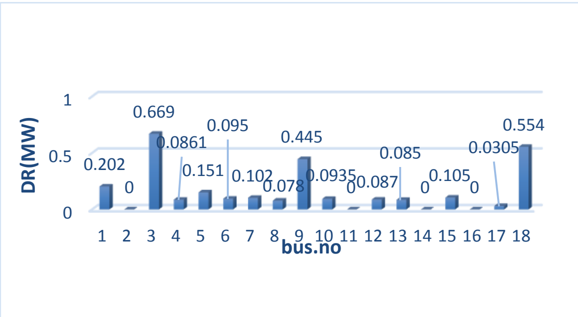


Fig. 3. Quota of existing customers in total load curtailment amount at hour 20 (Inelastic load)

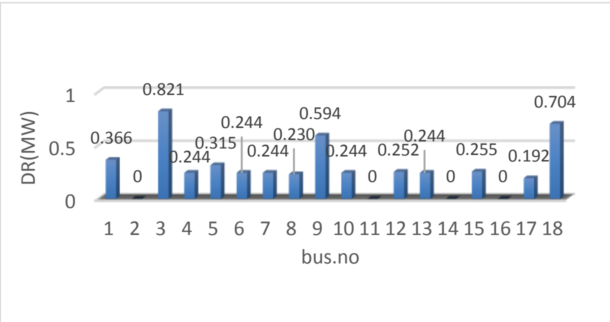


Fig. 4. Quota of existing customers in total load curtailment amount at hour 20 (Elastic load)

Similarly, Fig. 4 illustrates customers' reduction quotas at hour 20 in case of elastic load modeling. As it is shown, in this case maximum permissible load curtailment has been applied to each subscriber (See Table 1).

In above mentioned cases, contributions of existing DG units are shown in Table. 4. As illustrated, DG generation outputs are increased drastically in some hours (12, 13, 19- 21), when price of upstream market exceeds market selling price. At hour 12, DisCo utilizes its DG contribution to meet customers' load demand and to sell to the upstream market as well. However, during hours 13 and 19-21 it would be more beneficial to sell mostly to the upstream market rather than supplying local demand. Subsequently, relatively higher load curtailments occur within these periods. It is noticeable that generation of DG units in some other periods is due to higher wholesale market prices comparing their related selling prices. Obviously, quota of each DG contribution is based on its operation cost coefficient. It is worth mentioning that DG unit outputs are the same for both inelastic and elastic load modelings, since they are just dependant of upstream market prices and corresponding cost coefficients as well.

Table 4. Hourly DG contributions

h \ n	P1	P2	P3	P4	h \ n	P1	P2	P3	P4
1	0	0	0	0	13	4	5	5.5	7
2	0	0	0	0	14	4	0	5.5	7
3	0	0	0	0	15	4	0	5.5	7
4	0	0	0	0	16	4	0	5.5	7
5	0	0	0	0	17	4	0	5.5	7
6	0	0	0	0	18	4	0	5.5	7
7	4	0	0	0	19	4	5	5.5	7
8	4	0	0	0	20	4	5	5.5	7
9	4	0	0	0	21	4	5	5.5	7
10	4	0	5.5	0	22	4	0	5.5	7
11	4	0	5.5	7	23	4	0	0	0
12	4	5	5.5	7	24	4	0	0	0

Subsequently, load exchange of DisCo with upstream wholesale market is represented in Fig. 5. As shown, rate of purchased energy is decreased by executing DR programs in both elastic and inelastic load modelings. In addition, DisCo sells back its excessive power to upstream market while wholesale price is higher than market selling price. Nevertheless, it intends to purchase from wholesale market when corresponding upstream prices are relatively low. In this figure positive and negative values represent hourly purchased and sold powers, respectively. As it is appear, maximum export occurs at hour 13. However, unlike expectation, less export has been occured at hour 19 due to high existing demand in this period. As shown, both curves are almost coincident, except that DisCo decides to purchase less from and sell more to the upstream electricity market in case of elastic load modeling. This is due to much more load curtailments that would release DG capacities to sell back more excess power to the wholesale market.

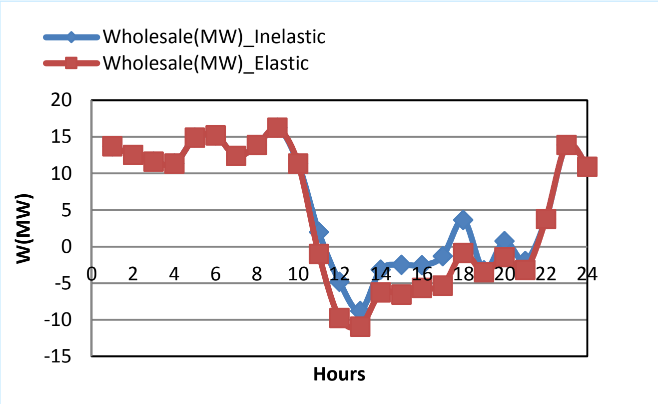


Fig. 5. Power exchange amounts with wholesale market

Based on load elasticity model, customers are exposed to much more load curtailments due to increased market prices. Considering corresponding load curtailment amounts (in Fig. 1), Table. 5 represents energy selling prices in presence and absence of DR programs. As shown, customers experience increases in energy prices in those periods that DR programs are applied. Obviously, no price variations will occur in case of inelastic load modeling.

Subsequently, DisCo's relative payoffs in three scenarios, entitled: "without DR", "with DR and inelastic load" and "with DR and elastic load" are represented in Table. 6. As can be seen, DisCo benefits more in case of elastic load modeling. It is noticeable that obtained payoffs are directly dependant on market prices and load curtailment amounts as well. Relatively high increases in payoffs at hour 13 are because of increasing in power sold to the upstream market. Note that profits are unchanged during first 10 hours in all scenarios, since no curtailment is implemented then (see Fig. 1). It is the same situation within first 12 hours in two scenarios "without DR" and "with DR and inelastic load".

Finally, impact of different load elasticity coefficients on DisCo's aggregated payoffs is illustrated in Fig. 6. Decreasing in load elasticity means to move towards inelastic load modeling. In this regard, corresponding payoffs decrease as load elasticity approaches to zero. Accordingly, one can conclude that when load elasticity becomes zero, obtained payoff would be exactly equal to DisCo's corresponding payoff in inelastic load modeling.

Table. 5. Energy selling prices with inelastic and elastic load models

Hour	Without DR ($/MWh)	With DR ($/MWh)	Hour	Without DR ($/MWh)	With DR ($/MWh)
1	38	38	13	67	69.14
2	38	38	14	60	61.96
3	38	38	15	60	61.99
4	38	38	16	60	61.88
5	38	38	17	60	61.85
6	38	38	18	60	61.44
7	60	60	19	67	68.45
8	60	60	20	67	68.32
9	60	60	21	67	68.43
10	60	60	22	67	67
11	67	68.09	23	60	60
12	67	68.98	24	60	60

Table. 6. DisCo's payoffs in different cases

Hour	Without DR($/MWh)	Inelastic load($/MWh)	Elastic load($/MWh)	Hour	Without DR ($/MWh)	Inelastic load($/MWh)	Elastic load($/MWh)
1	109.52	109.52	109.5	13	201.67	202.08	224.40
2	112.50	112.50	112.50	14	65.79	65.970	85.47
3	104.46	104.46	104.46	15	66.34	66.38	85.34
4	107.44	107.44	107.44	16	64.36	64.54	84.41
5	89.28	89.28	89.28	17	63.60	63.64	83.33
6	91.07	91.07	91.071	18	53.53	53.54	75.01
7	101.07	101.07	101.07	19	182.79	183.95	209.95
8	108.51	108.51	108.51	20	177.86	178.28	204.64
9	120.42	120.42	120.42	21	182.16	182.89	208.92
10	110.40	110.40	110.40	22	195.61	195.61	195.61
11	182.02	182.02	185.76	23	108.51	108.51	108.51
12	186.61	186.61	197.72	24	104.51	104.51	104.51

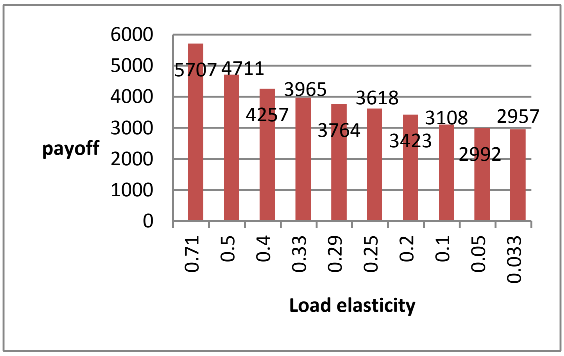


Fig. 6. Impact of different load elasticity coefficients on payoff

In order to validate the proposed model, the results were obtained by means of GAMS/CPLEX solver. Table 7, shows customers' hourly load curtailments for inelastic and elastic load modelings obtained from GAMS programming. Comparing the results with those obtained by the proposed mathematical modeling (in Fig.1), one can see an appropriate coincidence that validates the represented modeling.

Table 7. Customers' hourly load curtailments

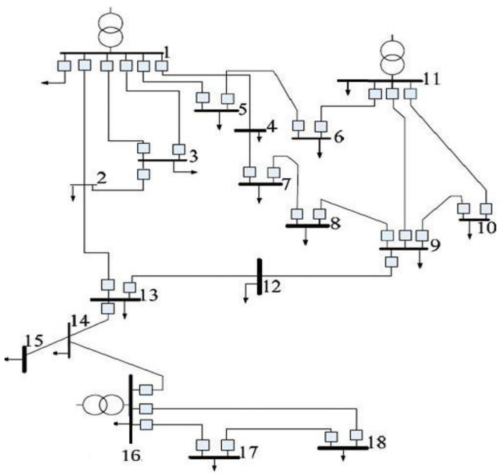
h / RD (MW)	1	2	3	4	5	6	7	8	9	10	11	12
Inelastic	0	0	0	0	0	0	0	0	0	0	0	0
Elastic	0	0	0	0	0	0	0	0	0	0	3.0	4.9
h / RD (MW)	13	14	15	16	17	18	19	20	21	22	23	24
Inelastic	2.7	1.8	0.9	1.8	0.9	0.4	4.6	2.7	3.7	0	0	0
Elastic	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	0	0	0

4. CONCLUSIONS

A heuristic mathematical modeling for DisCo's optimal decision-making is presented in which elastic and inelastic responsive load modelings as well as effect of owned DG contributions are taken into account. Furthermore, impact of customers' preferences in corresponding load curtailments is investigated by heuristic DR cost coefficients, which represent customers' reactions to aggregated load reduction amounts. The main innovation of this study in comparison to similar ones is presenting a novel mathematical framework for DisCo's optimal decision making rather than employing optimization algorithms. Based on proposed combinatorial framework, DisCo would obtain its: optimal trading with upstream market, optimal load curtailment amounts, DR incentives, and its appropriate DG contributions. As shown, in both elastic and inelastic load modelings, customers may experience load curtailments in those periods that are not profitable for DisCo to purchase from wholesale market. However, in case of elastic loads they would expose more curtailments due to increased market prices. In this case, despite much more payable incentive prices, DisCo prefers to make use of released DG capacities for selling back to the external grid. Accordingly, there would be some increases in DisCo's corresponding payoffs in comparison to inelastic load modeling. Also, it is shown that by moving load elasticity coefficients towards zero, the payoffs in both elastic and inelastic load modeling would be the same. Finally, comparing the results of proposed mathematical modeling with those obtained from GAMS/CPLEX solver, shows an appropriate coincidence.

Appendix:

IEEE 18 bus test system



From	To	R (Ω)	X (Ω)
1	2	0.07	0.15
1	3	0.03	0.07
1	4	0.09	0.21
1	5	0.03	0.08
2	3	0.01	0.02
2	13	0.12	0.18
4	7	0.03	0.07
5	6	0.08	0.19
6	11	0.09	0.2
7	8	0.06	0.13
8	9	0.11	0.22
9	10	0.22	0.2
9	12	0.1	0.2
10	12	0.12	0.26
12	13	0.12	0.18
13	14	0.19	0.33
14	15	0.25	0.38
14	16	0.11	0.21
16	17	0.22	0.42
16	18	0.32	0.6
17	18	0.24	0.45

REFERENCES

[1] S. Yousefi, M. P. Moghaddam and V. JohariMajd "Optimal real time pricing in an agent-based retail market using a comprehensive demand response model," *Energy*, vol. 36, no. 9, pp. 16-27, 2011

[2] H.A. Aalami, M.P. Moghaddam and G.R. Yousefi "Modeling and prioritizing demand response programs in power markets," *Electr. Power Syst. Res.*, vol. 80, no. 4, pp.426-435, 2011.

[3] M. Kazemi, A. Zangeneh and A. Badri "Prioritization of demand response programs in electricity power markets using TOPSIS," *Proc. Smart Grid Conf.*, pp. 327-333, 2013.

[4] H. Arasteh M.P. Moghaddam M, Sheikh El Eslami, and et al "Integrating commercial demand response resources with unit commitment," *Electr. Power Energy Syst.*, vol. 51, no. 1, pp. 153-161, 2013.

[5] A. Abdollahi, M.P. Moghaddam, M, Rashidinejad and et al "Investigation of economic & environmental driven demand response measures incorporating UC," *IEEE Trans. Smart Grid*, vol. 3, no. 1, pp. 12–25, 2012.

[6] C. S. Kirschen and D. Quantifying "The effect of demand response on electricity markets," *IEEE Trans. Power Syst.*, vol. 24, no. 1, pp. 199-207, 2009.

[7] H.A.Aalami, M.P. Moghaddam and G.R.Yousefi "Demand response modeling considering interruptible /curtailable loads and capacity market programs," *Appl. Energy*, vol. 87, no. 1, pp. 243-250, 2010.

[8] D. Nguen and H. Nguen and L. Le "Dynamic pricing design for demand response integration in power distribution networks ," *IEEE Trans. Power Syst.*, vol. 31, no. 5, pp. 3457-3472, 2016.

[9] F Meng and X. Zend , "A profit maximization approach to demand response management with customers behavior learning in smart grid," *IEEE Trans. Power Syst.*, vol. 7, no. 3, pp. 1516-1529, 2016.

[10] M. R. Sahebi, E. AbediniDuki, M. Kia, A. Soroudi and M. Ehsan, "Simultanous EDRP and unit commitment programming in comparison with interruptible load contracts", *IET Gener. Trans. Distrib.*, vol.6, no.7, pp. 605–611, 2012.

[11] H. Aalami, M. P. Moghadam, and G. R. Yousefi, "Determination of optimal demand response incentives using DR programs", *Proc. 22nd Int. Power Syst. Conf.*, pp. 132-136, 2007.

[12] N. Zareen, M. W. Mustafa, U. Sultana and etal, "Optimal real time cost benefit based demand response with intermittent resources", *Energy*, vol. 90, no.2, pp.1695-1706, 2015.

[13] Y. Wang, and M. Li, Lin, "Time of use based electricity demand response for sustainable manufacturing systems", *Energy*, vol. 63, no. 15, pp.233-244, 2013.

[14] M. Sarker, M. Vazquez and D.S. Kirschen "Optimal coordination and scheduling of demand response via monetary incentives," *IEEE Trans. Power Syst.*, vol. 6, no. 5, pp. 1341-1352, 2015.

[15] A. Badri, and K. Hosseinpour, "A stochastic multi period decision making framework of an electricity retailer considering aggregated optimal charging and discharging of electric vehicles", *J. Autom. Oper. Power Eng.*, vol. 3, no. 1, pp. 34-46, 2015.

[16] E. Bompard, R. Napoli, and B. Wan, "The effect of programs for demand response incentives in competitive electricity markets", *Eur. Trans. Electr. Power*, vol. 19, no. 1, pp.127-139, 2009.

[17] N. Ruiz, B. Claessens, J. Jimeno, and etal, "Residential load forecasting under a demand response program based on economic incentives," *Int. Trans. Electr. Energy Syst.*, vol. 25, no. 8, pp.1436-1451, 2015.

[18] H. Arasteh, M. Sepasian, and V. Vahidinasab, "Toward a smart distribution system expansion planning by considering demand response resources", *J. Autom. Oper. Power Eng.*, vol. 3, no. 2, pp. 116-130, 2015.

[19] M.Peik-Herfeh, H. Seifi and M.K. Sheikh-El-Eslami, "Decision making of a virtual power plant under uncertainties for bidding in a day-ahead market using point estimate method", *Electr. Power Energy Syst.*, vol. 44, no. 1, pp. 88-98, 2013.

[20] H. Kwag and J. Kim, "Optimal combined scheduling of generation and demand response with demand resource constraints", *Appl. Energy*, vol. 96, no. 2, pp.161-170, 2012.

[21] H. Aalami, G. R. Yousefi and M. P. Moghadam, "Demand response model considering EDRP and TOU programs", *Proc. IEEE/PES Transm. Distrib. Conf. Exhibition*, 2008.

[22] F. C. Schweppe, M. C. Caramanis, R. D. Tabors and R. E. Bohn, "Spot Pricing of Electricity", Boston, MA: Kluwer Academic Publishers, 1998.